Digital Signal Processing

Chapter 2 : Discrete Time Signal and Systems (Part 1)

Q. 1. Define terms 'signal' and 'system'?

Ans. A 'signal' may be defined as a physical quantity which varies with time, space or any independent variable Example — voltage, current A 'system may be defined as a combination of devices and networks or subsystem chosen to do a desired action Example Electrical N/W, mechanical system

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Q 2 Write the major classification of signals'

Ans. There are various types of signals Every signal is having its own characteristic The processing of signal mainly depends on the characteristics of that particular signal So classification of signal is necessary Broadly the signal are classified as follows

1 Continuous and discrete time signals

2. Continuous valued and discrete valued signals.

3. Periodic and non periodic signals.

4 Even and odd signals

5. Energy and power signals:

6 Deterministic and random signals

7. Multichannel and multidimensional signals.

Q. 3. Explain sampling function of sinc function.

Ans In mathematics, the sinc, function, denoted by sinc(x) and sometimes as Sa (x), has two definitions, In digital processing and information theory, the normalized sinc function is commonly defined by

$$\mathrm{Sa}(x) = \mathrm{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$$

In mathemetics, the historical unnormalized sinc function is defined by

$$\mathrm{Sa}(x) = \mathrm{sinc}(x) = \frac{\sin(x)}{x}$$

In both cases, the value of the function at the removable singularity at zero, usually calculated by l'Hospital rule, is something specified explicitly as the limit value 1 The sinc function is analytic everywhere.

Q. 4, What are energy and power signals?

Ans. The energy E of a signal x(n) is defined as

$$\mathrm{E} = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The energy of a signal can be finite or infinite. If E is finite $(i.e.\ 0 < \mathrm{E} < \infty)$, then x(n) is called an energy signal.

Many signals that posses infinite energy, have a finite average power. The average power of a discrete time signal x(n) is defined as

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} [x(n)]^2$$

If E is finite, P = 0. On the other hand, If E is infinite, the average power may be either finite or infinite. If P is finite (and non zero), the signal is called a power signal.

Q. 5. Differentiate between linear-Nonlinear system.

Ans. A system is called linear, if superposition principle applies to that system. This means that linear system may be defined as one whose response to the sum of the weighted inputs is same as the sum of the weighted responses.
Let us consider two systems defined as follows.

$$y_1(t) = f(x_1(t)) \qquad \text{...(1)}$$

Here x1(t) is the input or excitation and y1(t) is its output or response and

$$y_2(t) = f(x_2(t))$$

Here x2 (t) is the input or excitation and y2(t) is its output or response
Then for a linear system

$$f(a_1 x_1(t) + a_2 x_2(t)) = a_1 y_1(t) + a_2 y_2(t)$$

Where a1 and a2 are constants.
Linearity property for both continuous time and discrete time systems may be written as for continuous time system

$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 y_1(t) + a_2 y_2(t) \qquad \text{...(3)}$$

For discrete time system

$$a_1 x_1(n) + a_2 x_2(n) \longrightarrow a_1 y_1(n) + a_2 y_2(n) \qquad \text{...(4)}$$

For any non-linear system, the principle of super-position does not hold true and equations (3) and (4) are not satisfied.
Few examples of linear system are filters, communication channels etc.

Q. 6. Define periodic and non periodic signals Give an example in each case.

Ans A periodic signal repeats after fixed period But non-periodic signal never repeats Periodic signal like x(t) sin wt and Non periodic signal like $x(t) = e^{2t}$. A discrete time signal is periodic, if its frequency can be expressed as a ratio of two integers i.e.

$$f_0 = \frac{k}{N}$$

Here k and N are integer and N is the period of discrete time signal

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Q 7 What is scaling of discrete time signals?

Ans Scaling of discrete time signals is divided into two parts
Time Scaling Operations
As the name indicates, time scaling operations are related to the change in time scale. There are two types of time scaling operations.
• Down scaling (Compression)
Up scaling (Expansion)
(n) Amplitude Scaling Operation
As the name indicates, in case of amplitude scaling operations, amplitude of signal is changed Different amplitude scaling operations are as follows
• Upscaling (Amplification)
• Downscahng (Attenuation)
• Addition
• Multiplication

Q 8 What is the difference between static and dynamic discrete time signals?

Ans. There can be static and dynamic discrete time systems but cannot be signals

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Q 9 Define a discrete time unit sequence functions

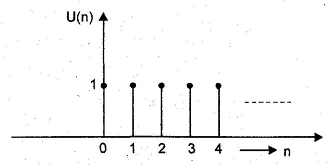
Ans. A discrete time unit signal is denoted by U(n) Its value is unity for all positive values of n. That means its value is one for n 0. While for other values of n, its value is zero.

$$U(n) = \begin{cases} 1 & for\, n \geq 0 \\ 0 & for\, n < 0 \end{cases}$$

In form of sequence it can be written as

$U(n) = \{1, 1, 1, 1, 1 \ldots\ldots\}$

Graphically it is represented as shown below

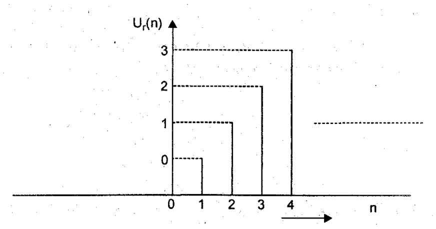


Q. 10. Define a discrete time unit ramp function.

Ans. A discrete time unit ramp function is denoted as Ur (n) and it is defined as

$$U_r(n) = \begin{cases} n \text{ for } n \geq 0 \\ 0 \text{ for } n < 0 \end{cases}$$

Figure below shows the graphical representation of a discrete unit ramp function.



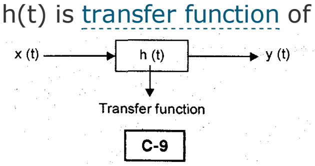
Q. 11. Define transfer function of a system.

Ans. A system may be defined as a set of elements or functional blocks which are connected together and produces an output in response to an input signal. The response of the system depends upon transfer function of the system.
Mathematically it is defined by

$$h(t) \text{ i.e. } h(t) = \frac{y(t)}{x(t)}$$

$$h(t) \text{ i.e. } h(t) = \frac{y(t)}{x(t)}$$

Where x(t) is input or excitation
y(t) is 0/P or response
h(t) is transfer function of the system.



Q. 12. State the necessary and sufficient condition for stability of LTI systems

Ans. LTI system is stable if its impulse response is absolutely summable i e

$$\sum_{k=-\infty}^{\infty} (h(k)) < \infty \qquad \ldots(1)$$

Here h(k)= h(n) is the impulse response of LTI system Thus equation (1) give the condition of stability in terms of impulse response of the system.

Now the stability factor is denoted by 's'.

$$s = \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Q. 13. Show that for LTI discrete-time system to stable, all the poles should lie within the unit circle

Ans. Stability Criteria for a Discrete-Time LTI Systems : The stability of discrete- time LTI system is equivalent to its impulse response h(n) being absolutely summable,

i.e.

$$S = \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

In this case the DTFT of h(n) converges and the ROC of transfer function H(z) must include the unit circle (z) =1

A discrete-time LTI system is stable if and only if the ROC of its transfer function H(z) includes the unit circle z = 1

However it is perfectly possible for a system to be stable but not causal For causal systems, stability can easily be checked by examining the locations of poles in transfer function H(z) For a causal discrete-time LTI system with rational transfer function H(z), the ROC is outside the outmost pole. A causal discrete time LTI system with rational transfer function H(z) is stable if and only if all of the poles of H(z) lie inside the unitcircle IzI =1.

For example Check the stability of the causal discrete-time LTI system with transfer function as under:

$$\mathbf{H}(z) = \frac{1}{1 - Az^{-1}}$$

Note that the given transfer function

$$\mathbf{H}(z) = \frac{1}{1 - Az^{-1}}$$

has a pole at z A. Now, this system will be stable if its pole is inside the unit circle. Iz = 1, i.e., A <1.

Determine of Impulse Response h (n):

We know that

$$h(n) = \text{Inverse } z\text{-transform } \{H(z)\}$$

$$h(n) = Z^{-1}[H(z)]$$

$$= Z^{-1}\left[\frac{1}{1 - Az^{-1}}\right]$$

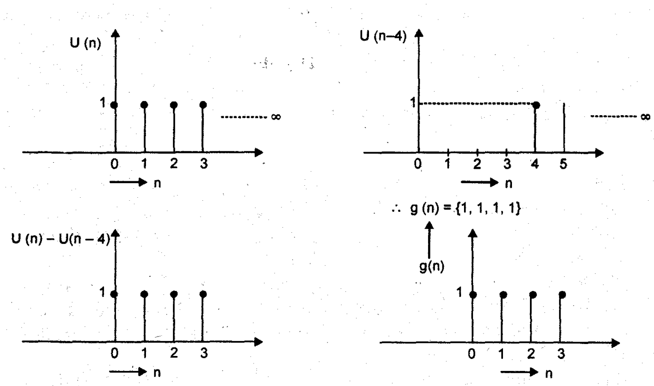
$$= A^n u(n)$$

$$h(n) = A^n u(n)$$

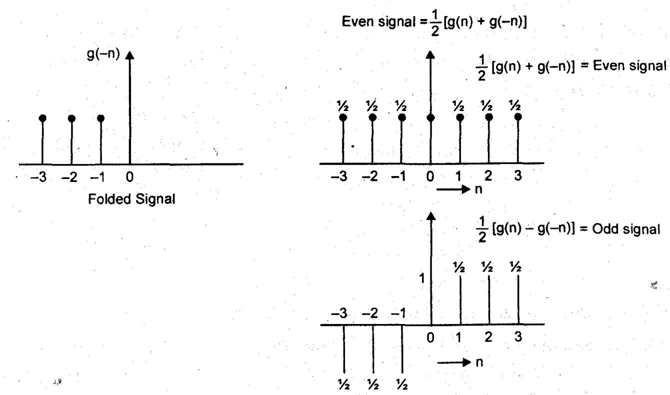
If h(n) is absolutely summable for I A I <1, then the system is stable.

Q. 14. Find the even and odd parts of the function g[n] = U [n] - U[n -4].

Ans.



The calculations of even and odd parts are shown in fig below



Q. 15. What are the constraints on the transfer function if it were to represent a causal LTI system?

Ans. If h(n) is the response of released LTI system to a unit impulse applied at n = 0, it follows that h(n) = 0 for n < 0 is both a necessary and a sufficient condition for causality Hence on LTI system is causal if and only if its impulse response is zero for negative values of n.

Q 16 Define LTI system'

Ans. If a system has both the linearity and time in varience properties, then the system is called as linear time m varient (LTI) system

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Q 17 What are the conditions for the region of convergence of a causal LTI system?

Ans. A discrete time LTI system is causal if and only if the ROC of its transfer function is the extension of a circle, in including infinite
A discrete time LTI systems which has a rational transfer function H(z) will be causal if and only if.
(z) The ROC is the extension of a circle outside the outermost pole and
(ii) Units H(z) expressed as a ratio of polynomials in z, the order of the numerator should be smaller than order of denomenator.

Q. 18. State sampling theorem.

Ans. A continuous time signal x(t) can be completely respresented in its sampled form and recoverd back from the sample form if the sampling frequency
$f_s \geq 2W$

where ‘W’ is the maximum frequency of the continuous time signal x(t)

Q. 19. Show that h (n) is equal to the convolution of the following signals.

$h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = (1/2)^n u(n).$

Ans.

$$h(n) = h_1(n) * h_2(n)$$

$$h(n) = (\delta(n) + \delta(n^{-1}) * \left(\frac{1}{2}\right)^n \mathrm{U}(x)$$

$$h_1(n) = \delta(n) + \delta(n-1)$$
$$h_1(0) = \delta(0) + \delta(0-1) = 1$$
$$h_1(1) = \delta(1) + \delta(1-1) = 1$$
$$h_2(2) = \delta(2) + \delta(2-1) = 0$$

$$h_1(n) = \{0, 0, \frac{1}{1}, 1, 0, 0 \text{ ....}, \infty\}$$

$$h_2(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$h_2(n) = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \text{......}, \infty\right\}$$

$$h(n) = h_1(n) * h_2(n)$$

$$= \{1, 1, 0, 0, 0 \text{ ....}\} * \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \text{......}\right\}$$

$$h(n) = \left\{1, 1, \frac{3}{4}, \frac{3}{8}, \frac{1}{8}, \text{......}\right\}$$

Q. 20. Convolve {1,3,1) and (1,2,2,).

Ans.

$$x(n) = \{1, 3, 1\}$$
$$h(n) = \{1, 2, 2\}$$

	$x(0)$ 1	$x(1)$ 3	$x(2)$ 1
$h(0)1$	1	3	1
$h(1)2$	2	6	2
$h(2)2$	2	6	2

Range of n is and

$$y_l = x_l + h_l = 0 + 0 = 0$$
$$y_h = x_h + h_h = 2 + 2 = 4$$
$$y(0) = h(0)\, x(0) = 1$$
$$y(1) = h(1)\, x(0) + h(0)\, x(1)$$
$$= 2 + 3 = 5$$

$$y(2) = h(2)\, x(0) + h(1)\, x(1) + h(0)\, x(2)$$
$$= 2 + 6 + 1 = 9$$
$$y(3) = h(2)\, x(1) + h(1)\, x(2)$$
$$= 6 + 2 = 8$$
$$y(4) = h(2)\, x(2) = 2$$
$$y(n) = \{1,5,9,8,2\}$$

y (n) is output of the convolution.

Q. 21. Compute convolution of y (n) of the signals.

$$\mathrm{X}(n) = \begin{cases} \alpha^n, -3 \le n \le 5 \\ 0, \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, 0 \le n \le 4 \\ 0, \text{elsewhere} \end{cases}$$

Ans.

$$X(n) = \begin{cases} \alpha^n, -3 \le n \le 5 \\ 0, \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, 0 \le n \le 4 \\ 0, \text{elsewhere} \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$x(n) = \left[\frac{1}{\alpha^3}, \frac{1}{\alpha^2}, \frac{1}{\alpha}, \underset{\uparrow}{1}, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5\right]$$

$$h(n) = \underset{\uparrow}{1}\,1\,1\,1\,1$$

$$y(n) = \sum_{k=-3}^{5} x(k)\, h(n-k)$$

$$y(0) = x(-3)\,h(3) + x(-2)\,h(2) + x(-1)\,h(1) + x(0)\,h(0) + x(1)\,h(-1) + x(2)\,h(-2) + x(3)\,h(-3) + x(4)\,h(-4) + x(5)\,h(-5)$$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1$$

$$y(1) = \sum_{k=-3}^{5} x(k) h(1-k)$$

$$= x(-3)\ h(4) + x(-2)\ h(3) + x(-1)\ h(2) + x(0)\ h(1) + x(1)\ h(0) + x(2)\ h(-1) + x(3)h(-2) + \ldots..$$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha$$

$$y(2) = \sum_{k=-3}^{5} x(k)\ h(2-k)$$

$$= x(-3)\,h(5) + x(-2)\,h(4) + x(-1)\,h(3) + x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0) + x(3)h(-1) + \ldots..$$

$$= \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha + \alpha^2$$

$$y(3) = \sum_{k=-3}^{5} x(k)h(3-k)$$

$$= x(-3)\,h(6) + x(-2)\,h(5) + x(-1)\,h(4) + x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1)\,x(3) + h(0) + x(4)\,h(-1) + x(5)\,h(-2)$$

$$= 0 + 0 + \frac{1}{\alpha} + 1 + \alpha + \alpha^2 + \alpha^3$$

$$= 1 + \alpha + \alpha^2 + \alpha^3 + \frac{1}{\alpha}$$

$$y(4) = \sum_{k=-3}^{5} x(k)h(4-k)$$

$$= x(-3)\,h(7) + x(-2)\,h(6) + x(-1)\,h(5) + x(0)\,h(4) + x(1)\,h(3) + x(2)\,h(2)\,x(3) + h(1) + x(4)\,h(0) + x(5)\,h(-1)$$

$$= 0 + 0 + 0 + 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4$$

$$= 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4$$

$$y(5) = \sum_{k=-3}^{5} x(k)n(5-k)$$

$$= x(-3)\,h(5) + x(-2)\,h(7) + x(-1)\,h(6) + x(0)\,h(5) + x(1)\,h(4) + x(2)\,h(3) + x(3)\,h(2) + x(4)\,h(1) + x(5)\,h(0)$$

$$= 0 + 0 + 0 + 0 + 1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$$

$$= 1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$$

$$y(6) = \sum_{k=-3}^{5} x(k)h(6-k)$$

$$= x(-3)\,h(9) + x(-2)\,h(8) + x(-1)\,h(7) + x(0)\,h(6) + x(1)\,h(5) + x(2)\,h(4) + x(3)\,h(3) + x(4)\,h(2) + x(5)\,h(1)$$

$$= 0 + 0 + 0 + 0 + 0 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$$

$$= \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$$

$$y(7) = \sum_{k=-3}^{5} x(k)h(7-k)$$

$$= x(-3)\,h(10) + x(-2)\,h(9) + x(-1)\,h(8) + x(0)\,h(7) + x(1)\,h(6) + x(2)\,h(5) + x(3)\,h(4) + x(4)\,h(3) + x(5)\,h(2)$$

$$= 0 + 0 + 0 + 0 + 0 + 0 + 0 + \alpha^3 + \alpha^4 + \alpha^5$$

$$= \alpha^3 + \alpha^4 + \alpha^5.$$

$$y(8) = \sum_{k=-3}^{5} x(k)h(8-k)$$

$$= x(-3)\,h(11) + x(-2)\,h(10) + x(-1)\,h(9) + x(0)\,h(8) + x(1)\,h(7) + x(2)\,h(6)\,x(3)\,h(5) + x(4)\,h(4) + x(5)\,h(3)$$

$$= 0 + 0 + 0 + 0 + 0 + 0 + \alpha^4 + \alpha^5 = \alpha^4 + \alpha^5$$

$$y(9) = \sum_{k=-3}^{5} x(k)h(9-k)$$

$$= x(-3)\,h(12) + x(-2)\,h(11) + x(-1)\,h(10) + x(0)\,h(9) + x(1)\,h(8) + x(2)\,h(7) + x(3) + h(6) + x(4)\,h(5) + x(5)\,h(4)$$

$$= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + \alpha^5 = \alpha^5.$$

$$y(-1) = \sum_{k=-3}^{5} x(k)\,h(-1-k)$$

$$= x(-3)\;h(2) + x(-2)\;h(1) + x(-1)\;h(0) + x(0)\;h(-1) + x(1)\;h(-2) + x(1)\;h(-3) + \ldots\ldots$$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 0 = \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha}$$

$$y(-2) = \sum_{k=-3}^{5} x(k)\,h(-2-k)$$

$$= x(-3)\,h(1) + x(-2)\,h(0) + x(-1)\,h(-1) + x(0)\,h(-2) + x(1)\,h(-3) + \ldots$$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2}$$

$$y(-3) = \sum_{k=-3}^{5} x(k)\,h(-3-k)$$

$$= x(-3)\,h(0) + x(-2)\,h(-1) + x(-1)\,h(-2) + x(0)\,h(-3) + x(1)\,h(-4) + x\,(2)\,h(-5) + \ldots.$$

$$= \frac{1}{\alpha^3}$$

$$y(n) = \left(\frac{1}{\alpha^3}, \frac{1}{\alpha^3} + \frac{1}{\alpha^2}, \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha}, \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1, \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha,, \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha + \alpha^2, \frac{1}{\alpha} + 1 + \alpha + \alpha^2 + \alpha^3, 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4, 1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \alpha^3 + \alpha^4 + \alpha^5, \alpha^4 + \alpha^5, \alpha^5\right)$$

Q. 22. What is the difference between stable astable system?

Ans.

Stable system	Astable System
1. An initialy relexed system is BIBO stable if and only if every bounded input produces bounded output.	1. An initially relexed system is said to be unstable if bounded input produces unbounded output.
2. Stable system shows finite behaviour.	2. Unstable system shows Eratic and extreme behaviour
3. When stable system is practically implemented then it cause limited range output.	3. When unstable system is practically implemented then it cause overflow.

Q. 23. Differentiate time variant from time invariant system.

Ans. A system is called time invariant if its input output characteristics do not charge with time. A LTI discrete time system satisfies boths the linearity and the time invariance properties.

To test if any given systems is time invariant, first apply an arbitrary sequence x (n) and find y (n).

y (n) = T [x (n)]

Now delay the input sequence by k samples and find output sequence denote it as. y(n,k) T[x(n-k)]

Delay the output sequence by k samples denote it as

$y(n, k) = y(n-k)$

For all possible values of k, the systems is the invariant on the other hand

$y(n, k) \neq y(n-k)$

Even for one value of k, the system is time variant.

the output.

Even for one value of k, the system is time variant.

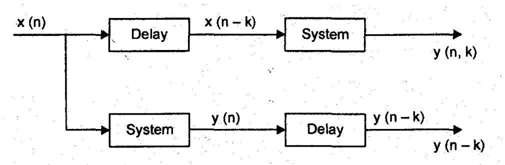


Fig. Time invariant and time variant system.

Q. 24. What are symmetric and asymmetric signals?

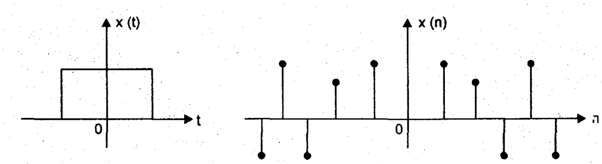
Ans. An even signal is that type of signal which exhibits symmetry in the time domain This type of signal is identical about the origin Mathematically, an even signal must satisfy the following condition.

For a continuous-time signal, x (t) = x (— t)

For a discrete-time signal, x (n) x (— n)

Figure shows continuous-time and discrete-time even signals.

Similarly, an odd signal is that type of signal which exhibits anti-symmetry. This type of signal is not identical about the origin Actually, the signal is identical to its negative Mathematically, an odd signal must satisfy the following condition



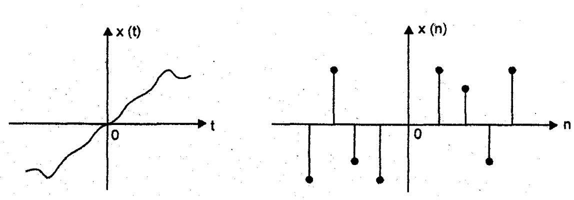
(a) Continuous-time even signal (b) Discrete-time even signal

Fig. Even signal

For a continuous-time signal, x (t) = x (- t)

For a discrete-time signal, x (n) — x (— n)

Figure shows continuous-time and discrete-time odd signals.



(a) Continuous-time odd signal (b) Discrete-time odd signal

Q. 25. What is the frequency response of a discrete LTI system? Derive the

frequency response of a system whose impulse response is given by h(n) = a” u(n —1) for (a) <1.

Ans. The frequency response of a linear time invariant discrete time system can be obtained by applying a spectrum of the input sinusoids to the system. The frequency response gives the gain and phase response of the system to the input sinusoids at all frequencies. Let us consider, the inpulse response of an LTI discrete time system is h(n) and the input x(n) to the system is complex exponential e1u. The output of the system y(n) can be

Given

$$h(n) = a^n U(n-1) \text{ for } |a| < 1$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h(n) e^{-jwn}$$

$$H(e^{jw}) = \sum_{n=0}^{\infty} a^n e^{-jwn} = \sum_{n=0}^{\infty} (ae^{-jw})^n$$

$$H(e^{j\omega}) = \frac{1}{1-ae^{-jw}}$$

Q. 26. Define the stability conditions for a linear time invarient system. Determine the range of values of ‘a’ for which the LTI system with impulse response h(n) as defined below is stable.

$$h(n) = a^n \quad n \geq 0, n \text{ even}$$
$$= 0 \text{ elsewhere}$$

(Dec, 2006)

Ans. LTl system is stable if its inpulse response is absolutely summable.

Consider a linear time invarient system having impulse response h(n). Let x(n) be input applied to this system. Now according to the definition of convolution, the output of such system is expressed as

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \quad \text{...(1)}$$

The input x(n) will be bounded if I x(n) is less than some finite number. Let us denote this finite number by M_X. Thus for input signal to be bounded

$$|x(n)| \leq M_x \quad \text{...(2)}$$

Here M_X is finite number, so its values should be less than infinity. Thus eq. (2) can be written as

$$|x(n)| \leq M_x < \infty \quad \text{...(3)}$$

Now taking absolute value of both sides of eq. (1)

$$|y(n)| = \left|\sum_{k=-\infty}^{\infty} h(k)x(n-k)\right| \quad \text{...(4)}$$

We will read R.H.S. of eq. (4) as absolute value of summation of terms. If we take ‘Σ’ sign outside then the term become

$$\sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|.$$

This summation of absolute

Thus:

values of terms. Always absolute values of sum of terms is less than or equal to the sum of absolute value of terms.

$$\left|\sum_{k=-\infty}^{\infty} h(k)x(n-k)\right| \leq \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)| \quad \text{...(5)}$$

But from eq. (4) the L.H.S term is $|y(n)|$

$$\therefore \quad y(n) \le \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)| \quad \text{...(6)}$$

Here x(n — k) is delayed input signal. If input is bounded then its delayed version is also bounded. This is because delay or qlding is related to time shifting operations By performing these operations the magrjitu4e is not changed. Now for bounded input,

we have

$$|x(n)| \le M_x$$

$$\therefore \quad |x(n-k)| \le M_x \quad \text{...(7)}$$

Putting this value in eq. 6, we get

$$|y(n)| \le \sum_{k=-\infty}^{\infty} |h(k)|\,M_x$$

$$\therefore \quad |y(n)| \le Mx \sum_{k=-\infty}^{\infty} |h(k)| \quad \text{...(8)}$$

We know that Mx is finite number. We want 0/P y(n) to be bounded. That means I y(n) should be finite. So eq. (8) to obtain finite output we have

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty \quad \text{...(9)}$$

Here h(k)= h(n) is the impulse response of LTI system. Thus eq. (9) gives the conditions of stability in terms of impulse response of the system.

Given that $h(n) = a^n\, U(n)$ n is even

This mean that $h(k) = a^k.\, U(k)$ so k is also even

We have $\sum_{k=0}^{\infty} |h(k)| = |a^k| = |a^0| + |a^2| + |a^4| + \text{........} + |a^k|$

From above, it is abvious that given system is stable if a <1.

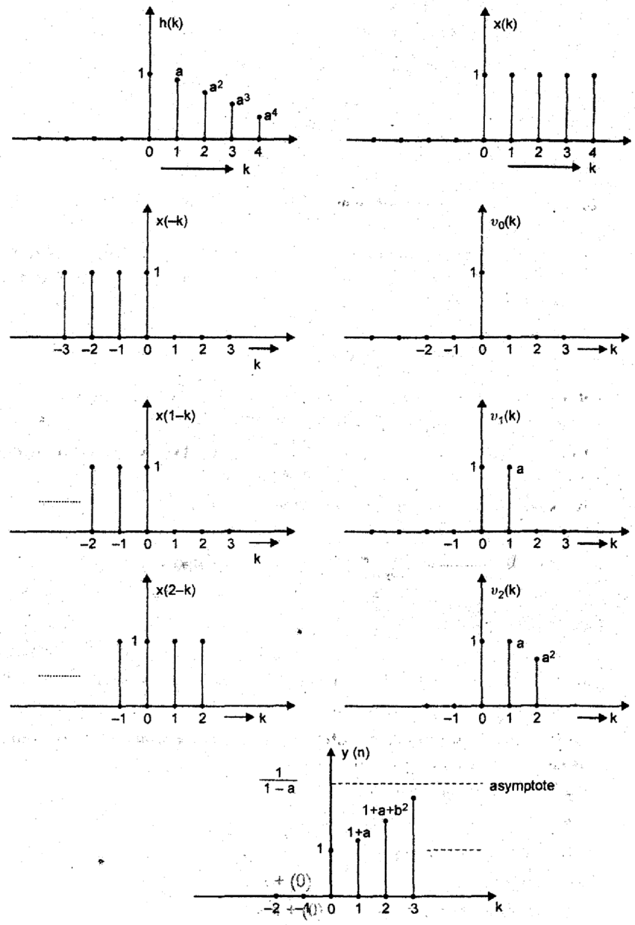
Q. 27. Determine the output y(n) of linear time invariant system units impulse response,
$h(n) = a^n\, U(n),\ |a| < 1$ **when the input is a unit step sequence x(n) = U(n).**

Ans. In this case bth h(n) and x (n) are infinite duration sequences. We use the form of the convolution formula

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

in which $x(k)$ is folded. The sequences $h(k)$, $x(k)$ and $x(-k)$ are shown in fig. The product of sequence $v_0(k)$, $v_1(k)$ and $v_2(k)$ corresponding to $x(-k)\ h(k)$, $x(1-k)\ h(k)$ and

x(2—k) h(k) are also illustrated in Fig.



Thus the outputs are

$$y(0) = 1$$
$$y(1) = 1 + a$$
$$y(2) = 1 + a + a^2$$

for $n > 0$, the output is

$$y(n) = 1 + a + a^2 + \ldots\ldots\ldots\ldots + a^n$$
$$= \frac{1 - a^{n+1}}{1 - a}$$

for $n < 0$, the product sequences consist of all zeroes. Hence

$$y(n) = 0\,;\; n < 0$$

A graph of output $y(n)$ is also illustrated in Fig for case $0 < a < 1$. Note the exponential in the output as a function of n. Since $|a| < 1$, the final value of the output as n approaches infinity is

$$y(\infty) = \lim_{n\to\infty} y(n) = \frac{1}{1-a}$$